



Alignment Document

State of Michigan And Aventa Learning Geometry

Geometry 2005-2007 Benchmark Blueprint

State Standard Number	State Standard Area / Description	Unit Name	Course Topic Description
1	Quantitative Literacy and Logic		
L1	Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.		
L1.1	Number Systems and Number Sense		
L1.1.1	Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.	Language of Geometry	Connections from Algebra
		Properties From Algebra and Proof	Reasoning and Introduction to Proof
L1.1.2	Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.		
L1.1.3	Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.	Commutative Properties	Connections From Algebra

L1.1.4	Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.		
L1.1.5	Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as $3m+k$, where k is 0, 1, or 2, and m is an integer; or that the sum of the first n positive integers is $n(n+1)/2$).		
L1.1.6	Explain the importance of the irrational numbers square root of 2 and square root of 3 in basic right triangle trigonometry; the importance of pi because of its role in circle relationships; and the role of e in applications such as continuously compounded interest.		
L1.2	Representations and Relationships		
L1.2.1	Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.		
L1.2.2	Interpret representations that reflect absolute value relationships (e.g. $ x - a $ "is less than or equal to" b , or $a \pm b$) in such contexts as error tolerance.		
L1.2.3	Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.		
L1.2.4	Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.		
L1.2.5	Read and interpret representations from various technological sources, such as contour or isobar diagrams.		

L1.3	Counting and Probabilistic Reasoning		
L1.3.1	Describe, explain, and apply various counting techniques (e.g., finding the number of different 4-letter passwords; permutations; and combinations); relate combinations to Pascal's triangle; know when to use each technique.		
L1.3.2	Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).		
L1.3.3	Recognize and explain common probability misconceptions such as "hot streaks" and "being due."		
L2	Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations.) They understand the basic ideas of iteration and algorithms.		
L2.1	Calculation Using Real and Complex Numbers		
L2.1.1	Explain the meaning and uses of weighted averages (e.g., GNP, consumer price index, grade point average).		
L2.1.2	Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.		
L2.1.3	Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.		
L2.1.4	Know that the complex number i is one of two solutions to $x^2 = -1$.		
L2.1.5	Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.		

L2.1.6	Recognize when exact answers aren't always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).		
L2.1.7	Understand the mathematical bases for the differences among voting procedures.		
L2.2	Sequences and Iteration		
L2.2.1	Find the n th term in arithmetic, geometric, or other simple sequences.		
L2.2.2	Compute sums of finite arithmetic and geometric sequences.		
L2.2.3	Use iterative processes in such examples as computing compound interest or applying approximation procedures.		
L2.2.4	Compute sums of infinite geometric sequences.		
L3	Students apply measurement units and calculations, and understand the concept of error.		
L3.1	Measurement Units, Calculations, and Scales		
L3.1.1	Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.		
L3.1.2	Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements (e.g., explain why a small change in the scale can represent a large change in intensity); solve applied problems.		
L3.2	Understanding Error		
L3.2.1	Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.		

L3.2.2	Describe and explain round-off error, rounding, and truncating.		
L3.2.3	Know the meaning of and interpret statistical significance, margin of error, and confidence level.		
L4	Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.		
L4.1	Mathematical Reasoning		
L4.1.1	Distinguish between inductive and deductive reasoning, identifying and providing examples of each.	Reasoning and Introduction to Proof	Deductive Reasoning
		Reasoning and Introduction to Proof	Inductive Reasoning
L4.1.2	Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.	Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
L4.1.3	Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics; identify and give examples of each.	Reasoning and Introduction to Proof	Inductive Reasoning
		Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
		Reasoning and Introduction to Proof	Deductive Reasoning
		Reasoning and Introduction to Proof	Two Column Proof with Segments and Angles
		Reasoning and Introduction to Proof	Properties From Algebra and Proof

L4.2	Language and Laws of Logic		
L4.2.1	Know and use the terms of basic logic (e.g., proposition, negation, truth and falsity, implication, if and only if, contrapositive, and converse).	If-Then Statements, Converses, and Postulates	Reasoning and Introduction to Proof
L4.2.2	Use the connectives "NOT," "AND," "OR," and "IF...,THEN," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.	Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
L4.2.3	Use the quantifiers "THERE EXISTS" and "ALL" in mathematical and everyday settings and know how to logically negate statements involving them.	Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
L4.2.4	Write the converse, inverse, and contrapositive of an "If..., then..." statement; use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.	Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
L4.3	Proof		
L4.3.1	Know the basic structure for the proof of an "If..., then..." statement (assuming the hypothesis and ending with the conclusion) and know that proving the contrapositive is equivalent.	Reasoning and Introduction to Proof	If-Then Statements, Converses, and Postulates
L4.3.2	Construct proofs by contradiction; use counterexamples, when appropriate, to disprove a statement.	Reasoning and Introduction to Proof	Reasoning and Introduction to Proof
L4.3.3	Explain the difference between a necessary and a sufficient condition within the statement of a theorem; determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.		

2	Algebra & Functions		
A1	Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.		
A1.1	Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A1.1.1	Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.	Concept of "Variable" in Geometry	Connections From Algebra
A1.1.2	Know the definitions and properties of exponents and roots, transition fluently between them, and apply them in algebraic expressions.		
A1.1.3	Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).		
A1.1.4	Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., multiply $(x - 1)(1 - x^2 + 3)$; simplify $(9x - x^3)/(x + 3)$)		
A1.1.5	Divide a polynomial by a monomial.		
A1.1.6	Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.		

A1.1.7	Transform trigonometric expressions into equivalent forms using basic identities such as: $\sin^2 \theta + \cos^2 \theta = 1$, $\tan \theta = \sin \theta / \cos \theta$ and $\tan^2 \theta + 1 = \sec^2 \theta$		
A1.2	Solutions of Equations and Inequalities (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A1.2.1	Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.	Parallel Lines and Coordinate Planes	Equations of lines
		Language of Geometry	Rays and Angles
		Language of Geometry	Pairs of Angles
		Language of Geometry	Right Angles and Perpendicular Lines
A1.2.1	Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.	Quadrilaterals and Polygons	Square and Rectangle
		Quadrilaterals and Polygons	Rhombus and Trapezoid
A1.2.2	Associate a given equation with a function whose zeros are the solutions of the equation.		
A1.2.3	Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.		
A1.2.4	Solve absolute value equations and inequalities, (e.g. solve $ x - 3 $ "is less than or equal to 6), and justify steps in the solution.	Language of Geometry	Measuring Segments
A1.2.5	Solve polynomial equations and equations involving rational expressions (e.g. solve $-2x(x^2 + 4x + 3) = 0$; solve $x - (1/(x + 6)) = 3$), and justify steps in the solution.		

A1.2.6	Solve power equations (e.g., $(x + 1)^3 = 8$) and equations including radical expressions (e.g., the square root of $(3x - 7) = 7$), justify steps in the solution, and explain how extraneous solutions may arise.		
A1.2.7	Solve exponential and logarithmic equations (e.g., $3(2 \text{ to the } x \text{ power}) = 24$, $2 \ln(x + 1) = 4$), and justify steps in the solution.		
A1.2.8	Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.		
A1.2.9	Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = velocity x time), and apply appropriately in contextual situations.	Lines and Points in Coordinate Plane	Parallel Lines and Coordinate Plane
		Parallel Lines and Coordinate Plane	Lines and Points in Coordinate Plane
		Parallel Lines and Coordinate Plane	Equations of Lines in Coordinate Plane
A1.2.10	Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals (e.g., $2\sin x - 1 = 0$ for $0 \leq x \leq 2\pi$).		
A2	Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.		
A2.1	Definitions, Representations, and Attributes of Functions		
A2.1.1	Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.		

A2.1.2	Read, interpret, and use function notation, and evaluate a function at a value in its domain.		
A2.1.3	Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.		
A2.1.4	Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined (e.g., absolute value and greatest integer functions).		
A2.1.5	Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions (e.g., $f(0) = 5$, and $f(n) = f(n-1) + 2$).		
A2.1.6	Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as x approaches positive or negative infinity, given the symbolic and graphical representations.		
A2.1.7	Identify and interpret the key features of a function from its graph or its formula(e), (e.g. slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, average rate of change over an interval, and periodicity).		
A2.2	Operations and Transformations		
A2.2.1	Combine functions by addition, subtraction, multiplication, and division.		
A2.2.2	Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the x - and y -axes) to basic functions, and represent symbolically.		
A2.2.3	Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs (e.g., $f(x) = x^3$ and $g(x) = x$ to the $1/3$ power).		

A2.2.4	If a function has an inverse, find the expression(s) for the inverse.		
A2.2.5	Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.		
A2.2.6	Know and interpret the function notation for inverses and verify that two functions are inverses using composition.		
A2.3	Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A2.3.1	Identify a function as a member of a family of functions based on its symbolic, or graphical representation; recognize that different families of functions have different asymptotic behavior at infinity, and describe these behaviors.		
A2.3.2	Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.		
A2.3.3	Write the general symbolic forms that characterize each family of functions. (e.g., $f(x) = A$ to the base 0 of a to the x power; $f(x) = A\sin Bx$)		
A2.4	Lines and Linear Functions		
A2.4.1	Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$, where B "is not equal to" 0], point-slope, and slope-intercept) given appropriate information, and convert between forms.	Parallel Lines and Coordinate Plane	Equations of Lines in Coordinate Plane
A2.4.2	Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.	Parallel Lines and Coordinate Plane	Equations of Lines in Coordinate Plane
A2.4.3	Relate the coefficients in a linear function to the slope and x- and y-intercepts of its graph.	Parallel Lines and Coordinate Plane	Equations of Lines in Coordinate Plane

A2.4.4	Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give -1.	Parallel Lines and Coordinate Plane	Equations of Lines in Coordinate Plane
A2.5	Exponential and Logarithmic Functions		
A2.5.1	Write the symbolic form and sketch the graph of an exponential function given appropriate information. (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4(1.5)^x$).		
A2.5.2	Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^x$, $f(x) = \log x$, $f(x) = e^x$, $f(x) = \ln x$); recognize the logarithmic function as the inverse of the exponential function.		
A2.5.3	Apply properties of exponential and logarithmic functions (e.g., $a^{x+y} = a^x a^y$, $\log(ab) = \log a + \log b$).		
A2.5.4	Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.		
A2.5.5	Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.		
A2.6	Quadratic Functions		
A2.6.1	Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).		
A2.6.2	Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.		

A2.6.3	Convert quadratic functions from standard to vertex form by completing the square.		
A2.6.4	Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.		
A2.6.5	Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.		
A2.7	Power Functions (including roots, cubics, quartics, etc.)		
A2.7.1	Write the symbolic form and sketch the graph of power functions.		
A2.7.2	Express direct and inverse relationships as functions (e.g., $y = kx$ to the n power and $y = kx$ to the $-n$ power, $n > 0$) and recognize their characteristics (e.g., in $y = x^3$, note that doubling x results in multiplying y by a factor of 8).		
A2.7.3	Analyze the graphs of power functions, noting reflectional or rotational symmetry.		
A2.8	Polynomial Functions		
A2.8.1	Write the symbolic form and sketch the graph of simple polynomial functions.		
A2.8.2	Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.		
A2.8.3	Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the x -intercepts of the graph and the factored form of the function.		
A2.9	Rational Functions		
A2.9.1	Write the symbolic form and sketch the graph of simple rational functions.		

A2.9.2	Analyze graphs of simple rational functions (e.g., $f(x) = (2x + 1)/(x - 1)$; $g(x) = x/(x^2 - 4)$) and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.		
A2.10	Trigonometric Functions		
A2.10.1	Use the unit circle to define sine and cosine; approximate values of sine and cosine (e.g., $\sin 3$, or $\cos 0.5$); use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.		
A2.10.2	Use the relationship between degree and radian measures to solve problems.		
A2.10.3	Use the unit circle to determine the exact values of sine and cosine, for integer multiples of $\pi/6$ and $\pi/4$.		
A2.10.4	Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.		
A2.10.5	Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.		
A3	Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.		
A3.1	Models of Real-world Situations Using Families of Functions.		

A3.1.1	Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. In the example above, recognize that the appropriate general function is exponential ($P = P_0 a^t$ to the base 0 of a to the t power)		
A3.1.2	Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. In the example above, substitute the given values $P_0 = 300$ and $a = 1.02$ to obtain $P = 300(1.02)^t$ to the t power.		
A3.1.3	Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.		
A3.1.4	Use methods of linear programming to represent and solve simple real-life problems.		
3	Geometry & Trigonometry		
G1	Students represent basic geometric figures, polygons, and conic sections and apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area, and use them to solve problems. They know and apply properties of common three-dimensional figures.		
G1.1	Lines and Angles; Basic Euclidean and Coordinate Geometry		

G1.1.1	Solve multi-step problems and construct proofs involving vertical angles, linear pairs of angles supplementary angles, complementary angles, and right angles.	Right Angles and Perpendicular Lines	Connections From Algebra
		Reasoning and Introduction to Proof	Reasoning and Introduction to Proof
		Classifying Angles	Connections From Algebra
		Two Column Proof With Segments and Angles	Reasoning and Introduction to Proof
		Pairs of Angles	Connections From Algebra
		Reasoning and Introduction to Proof	Reasoning and Introduction to Proof
G1.1.3	Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.	Bisecting a Segment	Connections From Algebra
		Rays and Angles	Connections From Algebra
G1.1.4	Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.	Right Angles and Perpendicular Lines	Connections From Algebra
		Lines and Points in Coordinate Plane	Parallel Lines and Coordinate Plane
G1.1.5	Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.	Parallel Lines and Coordinate Plane	Distance Formula
		Parallel Lines and Coordinate Plane	Midpoint Formula
G1.1.6	Recognize Euclidean Geometry as an axiom system; know the key axioms and understand the meaning of and distinguish between undefined terms (e.g., point, line, plane), axioms, definitions, and theorems.	Points	Connections From Algebra
		Ray	Connections From Algebra
G1.2	Triangles and Their Properties		
G1.2.1	Prove that the angle sum of a triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.	Basic Closed Figures in Geometry	Triangles: Basic Closed Figures in Geometry

G1.2.2	Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter, and area of all types of triangles.	Triangles and Special Relationships in Triangles	Right Triangles and the Pythagorean Theorem (also 30-60-90 and 45-45-90)
		Basic Closed Figures in Geometry	Triangles: Basic Closed Figures in Geometry
		Isosceles Triangles	Special Triangles and Special Relationships in Triangles
		Equilateral Triangles	Special Triangles and Special Relationships in Triangles
		Perimeters and Areas	Perimeters and Areas of Triangles and Polygons
G1.2.3	Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multi-step problems.	Right Triangles and Pythagorean Theorem	Special Triangles and Special Relationships in Triangles
G1.2.4	Prove and use the relationships among the side lengths and the angles of 30°-60°-90° triangles and 45°-45°-90° triangles.	Triangles and Special Relationships in Triangles	Right Triangles and the Pythagorean Theorem (also 30-60-90 and 45-45-90)
G1.2.5	Solve multi-step problems and construct proofs about the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; using a straightedge and compass, construct these lines.	Equilateral Triangles	Special Triangles and Special Relationships in Triangles
		Triangles and Special Relationships in Triangles	Isosceles Triangles
		Triangles Basic Closed Figures in Geometry	Special Segments in Triangles
		Rays and Angles	Connections From Algebra
		Special Segments in Triangles	Triangles: Basic Closed Figures in Geometry
		Language of Geometry	Constructing a segment bisector and an angle bisector
G1.3	Triangles and Trigonometry		
G1.3.1	Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.	Special Ratios in a Right Triangle	Right Triangle and Trigonometry

G1.3.2	Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides a and b and included angle theta using the formula Area = $(1/2) a b \sin \theta$.	Law of Sines and Cosines	Right Triangle and Trigonometry
G1.3.3	Determine the exact values of sine, cosine, and tangent for 0° , 30° , 45° , 60° , and their integer multiples, and apply in various contexts.		
G1.4	Quadrilaterals and Their Properties		
G1.4.1	Solve multi-step problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.	The Rhombus and Trapezoid	Quadrilaterals and Polygons
		Perimeters and Areas of Triangles and Polygons	Perimeters and Areas
		Squares and Rectangles	Quadrilaterals and Polygons
		Parallelograms	Quadrilaterals and Polygons
G1.4.2	Solve multi-step problems and construct proofs involving quadrilaterals (e.g., prove that the diagonals of a rhombus are perpendicular) using Euclidean methods or coordinate geometry.	Squares and Rectangles	Quadrilaterals and Polygons
G1.4.3	Describe and justify hierarchical relationships among quadrilaterals, (e.g. every rectangle is a parallelogram).	Perimeters and Areas of Quadrilaterals	Perimeters and Areas
		Squares and Rectangles	Quadrilaterals and Polygons
		Parallelograms	Quadrilaterals and Polygons
		The Rhombus and Trapezoid	Quadrilaterals and Polygons
G1.4.4	Prove theorems about the interior and exterior angle sums of a quadrilateral.	Parallelograms	Quadrilaterals and Polygons
		The Rhombus and Trapezoid	Quadrilaterals and Polygons
		Perimeters and Areas of Quadrilaterals	Perimeters and Areas
		Squares and Rectangles	Quadrilaterals and Polygons
G1.4.5	Understand the definition of a cyclic quadrilateral and know and use the basic properties of cyclic quadrilaterals.		

G1.5	Other Polygons and Their Properties		
G1.5.1	Know and use subdivision or circumscription methods to find areas of polygons (e.g., regular octagon, non-regular pentagon).	Perimeters and Areas of Triangles and Polygons	Perimeters and Areas
G1.6	Circles and Their Properties		
G1.6.1	Solve multi-step problems involving circumference and area of circles.	Circumferences and Areas of Circles	Perimeters and Areas
G1.6.2	Solve problems and justify arguments about chords (e.g., if a line through the center of a circle is perpendicular to a chord, it bisects the chord) and lines tangent to circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of tangency).	Arcs and Special segments	Circles
G1.6.3	Solve problems and justify arguments about central angles, inscribed angles and triangles in circles.		
G1.6.4	Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.	Arcs and Special segments	Circles
G1.7	Conic Sections and Their Properties		
G.1.7.1	Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.	Equations of Circles	Circles
G1.7.2	Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.		
G1.7.3	Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.		
G1.7.4	Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola; interpret these relationships in applied contexts.		
G1.8	Three- Dimensional Figures		
G1.8.1	Solve multi-step problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.		

G1.8.2	Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.		
G2	Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.		
G2.1	Relationships Between Area and Volume Formulas		
G2.1.1	Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.	Special Segments in Triangles	Triangles: Basic Closed Figures in Geometry
		Perimeters and Areas of Quadrilaterals	Perimeters and Areas
		Isosceles Triangles	Special Triangles and Special Relationships in Triangles
		Parallelograms	Quadrilaterals and Polygons
		Basic Closed Figures in Geometry	Triangles: Basic Closed Figures in Geometry
		The Rhombus and Trapezoid	Quadrilaterals and Polygons
G2.1.2	Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).		
G2.1.3	Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).		
G2.2	Relationships Between Two-dimensional and Three-dimensional Representations		
G2.2.1	Identify or sketch a possible 3-dimensional figure, given 2-dimensional views (e.g., nets, multiple views); create a 2-dimensional representation of a 3-dimensional figure.		

G2.2.2	Identify or sketch cross-sections of 3-dimensional figures; identify or sketch solids formed by revolving 2-dimensional figures around lines.		
G2.3	Congruence and Similarity		
G2.3.1	Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.	Congruent Triangles and Congruence Tests (excludes HL and AAS)	Triangles: Basic Closed Figures in Geometry
G2.3.2	Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.	Congruent Triangles and Congruence Tests	Triangles: Basic Closed Figures in Geometry
G2.3.3	Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.	Triangles Basic Closed Figures in Geometry	Congruent Triangles and Congruence Tests
G2.3.4	Use theorems about similar triangles to solve problems with and without use of coordinates.	Similarity Topic	Similar Triangles
G2.3.5	Know and apply the theorem stating that the effect of a scale factor of k relating one two dimensional figure to another or one three dimensional figure to another, on the length, area, and volume of the figures is to multiply each by k , k^2 , and k^3 , respectively.		
G3	Students will solve problems about distance-preserving transformations and shape-preserving transformations. The transformations will be described synthetically and, in simple cases, by analytic expressions in coordinates.		
G3.1	Distance-preserving Transformations: Isometries		
G3.1.1	Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.		
G3.1.2	Given two figures that are images of each other under an isometry, find the isometry and describe it completely.		

G3.1.3	Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.		
G3.2	Shape-preserving Transformations: Dilations and Isometries		
G3.2.1	Know the definition of dilation, and find the image of a figure under a given dilation.		
G3.2.2	Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.		
G3.2.3	Find the image of a figure under the composition of a dilation and an isometry.		
4	Statistics & Probability		
S1	Students plot and analyze univariate data by considering the shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.		
S1.1	Producing and Interpreting Plots		
S1.1.1	Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.		
S1.1.2	Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.		



S1.2	Measures of Center and Variation		
S1.2.1	Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.		
S1.2.2	Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.		
S1.2.3	Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.		
S1.3	The Normal Distribution		
S1.3.1	Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.		
S1.3.2	Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.		
S1.3.3	Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.		
S1.3.4	Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.		
S2	Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.		
S2.1	Scatterplots and Correlation		
S2.1.1	Construct a scatterplot for a bivariate data set with appropriate labels and scales.		

S2.1.2	Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.		
S2.1.3	Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.		
S2.1.4	Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.		
S2.2	Linear Regression		
S2.2.1	For bivariate data which appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line; interpret the slope of the equation for a regression line.		
S2.2.2	Use the equation of the least squares regression line to make appropriate predictions.		
S3	Students understand and apply sampling and various sampling methods, examine surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias. They understand basic principles of good experimental design.		
S3.1	Data Collection and Analysis		
S3.1.1	Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.		
S3.1.2	Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.		

S3.1.3	Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.		
S3.1.4	Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.		
S3.1.5	Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.		
S3.1.6	Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.		
S3.2.1	Explain the basic ideas of statistical process control, including recording data from a process over time.		
S3.2.2	Read and interpret basic control charts; detect patterns and departures from patterns.		
S4	Students understand probability and find probabilities in various situations, including those involving compound events, using diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.		
S4.1	Probability		
S4.1.1	Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).		
S4.1.2	Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.		



S4.1.3	Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.		
S4.2	Application and Representation		
S4.2.1	Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.		
S4.2.2	Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.		