



Alignment Document

State of Michigan And Aventa Learning Algebra 2

Algebra 2 2005-2007 Benchmark Blueprint

State Standard Number	State Standard Area / Description	Unit Name	Course Topic Description
1	Quantitative Literacy and Logic		
L1	Based on their knowledge of the properties of arithmetic, students understand and reason about numbers, number systems, and the relationships between them. They represent quantitative relationships using mathematical symbols, and interpret relationships from those representations.		
L1.1	Number Systems and Number Sense		
L1.1.1	Know the different properties that hold in different number systems, and recognize that the applicable properties change in the transition from the positive integers, to all integers, to the rational numbers, and to the real numbers.		
L1.1.2	Explain why the multiplicative inverse of a number has the same sign as the number, while the additive inverse of a number has the opposite sign.		
L1.1.3	Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.		
L1.1.4	Describe the reasons for the different effects of multiplication by, or exponentiation of, a positive number by a number less than 0, a number between 0 and 1, and a number greater than 1.		

L1.1.5	Justify numerical relationships (e.g., show that the sum of even integers is even; that every integer can be written as $3m+k$, where k is 0, 1, or 2, and m is an integer; or that the sum of the first n positive integers is $n(n+1)/2$).		
L1.1.6	Explain the importance of the irrational numbers square root of 2 and square root of 3 in basic right triangle trigonometry; the importance of pi because of its role in circle relationships; and the role of e in applications such as continuously compounded interest.		
L1.2	Representations and Relationships		
L1.2.1	Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.	Absolute Value	Shortcuts
L1.2.2	Interpret representations that reflect absolute value relationships (e.g. $ x - a $ "is less than or equal to" b , or $a \pm b$) in such contexts as error tolerance.	Absolute Value	Absolute Value and Inequalities
		Absolute Value	Shortcuts
L1.2.3	Use vectors to represent quantities that have magnitude and direction; interpret direction and magnitude of a vector numerically, and calculate the sum and difference of two vectors.		
L1.2.4	Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.		
L1.2.5	Read and interpret representations from various technological sources, such as contour or isobar diagrams.		
L1.3	Counting and Probabilistic Reasoning		

L1.3.1	Describe, explain, and apply various counting techniques (e.g., finding the number of different 4-letter passwords; permutations; and combinations); relate combinations to Pascal's triangle; know when to use each technique.	Counting	Factorial notation
		Counting	Counting: An introduction to the Multiplication Principle
		Counting	Combinations
		Counting	Some computations with factorials
		Counting	More about the Multiplication Principle
		Counting	Counting Subsets Formula
		Counting	Pascal's triangle
		Counting	Permutations
		Counting	The values in Pascal's triangle as factorials
		Counting	Counting: An introduction to choosing subsets
L1.3.2	Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).	Counting	Probability: An introduction
		Counting	Probability: More examples
		Counting	Frequency Expectation Interpretation of probability
		Counting	Introduction
L1.3.3	Recognize and explain common probability misconceptions such as "hot streaks" and "being due."	Counting	Frequency Expectation Interpretation of probability
L2	Students calculate fluently, estimate proficiently, and describe and use algorithms in appropriate situations (e.g., approximating solutions to equations.) They understand the basic ideas of iteration and algorithms.		
L2.1	Calculation Using Real and Complex Numbers		
L2.1.1	Explain the meaning and uses of weighted averages (e.g., GNP, consumer price index, grade point average).		

L2.1.2	Calculate fluently with numerical expressions involving exponents; use the rules of exponents; evaluate numerical expressions involving rational and negative exponents; transition easily between roots and exponents.		
L2.1.3	Explain the exponential relationship between a number and its base 10 logarithm, and use it to relate rules of logarithms to those of exponents in expressions involving numbers.	Exponential and Logarithm functions	Logarithm functions
		Exponential and Logarithm functions	Real Life Logarithmic Examples
		Exponential and Logarithm functions	Logarithm functions and exponential functions together
		Exponential and Logarithm functions	More Real Life Logarithmic Examples
		Exponential and Logarithm functions	Values of logarithm functions
		Exponential and Logarithm functions	Values of logarithm functions: a look at your calculator
L2.1.4	Know that the complex number i is one of two solutions to $x^2 = -1$.	Complex Numbers	The complex number i
		Complex Numbers	Working with complex numbers
L2.1.5	Add, subtract, and multiply complex numbers; use conjugates to simplify quotients of complex numbers.	Complex Numbers	Graphing Complex Numbers
		Complex Numbers	Addition and Subtraction in a Complex Plane
		Complex Numbers	Absolute Value of a Complex Number
		Complex Numbers	Working with complex numbers
L2.1.6	Recognize when exact answers aren't always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).	Quadratics	The Discriminant of a Quadratic
L2.1.7	Understand the mathematical bases for the differences among voting procedures.		



L2.2	Sequences and Iteration		
L2.2.1	Find the nth term in arithmetic, geometric, or other simple sequences.	Sequences and Series	Arithmetic Series
		Sequences and Series	Geometric Series
L2.2.2	Compute sums of finite arithmetic and geometric sequences.	Sequences and Series	Implicitly (or Recursively) defined sequences
		Sequences and Series	Series: An important example
		Sequences and Series	One very special Arithmetic Series
		Sequences and Series	Arithmetic sequences
		Sequences and Series	Summation notation (also called Sigma notation)
		Sequences and Series	Geometric Series
		Sequences and Series	Sequences
		Sequences and Series	Geometric sequences
		Sequences and Series	Sigma notation and series
		Sequences and Series	Explicitly defined sequences
		Sequences and Series	Series
		Sequences and Series	Arithmetic Series
L2.2.3	Use iterative processes in such examples as computing compound interest or applying approximation procedures.	Exponential and Logarithmic Functions	Exponential functions: An intuitive approach
L2.2.4	Compute sums of infinite geometric sequences.	Sequences and Series	Geometric sequences
L3	Students apply measurement units and calculations, and understand the concept of error.		
L3.1	Measurement Units, Calculations, and Scales		
L3.1.1	Convert units of measurement within and between systems; explain how arithmetic operations on measurements affect units, and carry units through calculations correctly.		

L3.1.2	Describe and interpret logarithmic relationships in such contexts as the Richter scale, the pH scale, or decibel measurements (e.g., explain why a small change in the scale can represent a large change in intensity); solve applied problems.	Exponential and Logarithm functions	Logarithm functions and exponential functions together
		Exponential and Logarithm functions	More Real Life Logarithmic Examples
		Exponential and Logarithm functions	Values of logarithm functions
		Exponential and Logarithm functions	Values of logarithm functions: a look at your calculator
		Exponential and Logarithm functions	Logarithm functions
		Exponential and Logarithm functions	Real Life Logarithmic Examples
L3.2	Understanding Error		
L3.2.1	Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.		
L3.2.2	Describe and explain round-off error, rounding, and truncating.		
L3.2.3	Know the meaning of and interpret statistical significance, margin of error, and confidence level.		
L4	Students understand mathematical reasoning as being grounded in logic and proof and can distinguish mathematical arguments from other types of arguments. They can interpret arguments made about quantitative situations in the popular media. Students know the language and laws of logic and can apply them in both mathematical and everyday settings. They write proofs using direct and indirect methods and use counterexamples appropriately to show that statements are false.		
L4.1	Mathematical Reasoning		

L4.1.1	Distinguish between inductive and deductive reasoning, identifying and providing examples of each.		
L4.1.2	Differentiate between statistical arguments (statements verified empirically using examples or data) and logical arguments based on the rules of logic.		
L4.1.3	Define and explain the roles of axioms (postulates), definitions, theorems, counterexamples, and proofs in the logical structure of mathematics; identify and give examples of each.		
L4.2	Language and Laws of Logic		
L4.2.1	Know and use the terms of basic logic (e.g., proposition, negation, truth and falsity, implication, if and only if, contrapositive, and converse).		
L4.2.2	Use the connectives "NOT," "AND," "OR," and "IF..., THEN," in mathematical and everyday settings. Know the truth table of each connective and how to logically negate statements involving these connectives.		
L4.2.3	Use the quantifiers "THERE EXISTS" and "ALL" in mathematical and everyday settings and know how to logically negate statements involving them.		
L4.2.4	Write the converse, inverse, and contrapositive of an "If..., then..." statement; use the fact, in mathematical and everyday settings, that the contrapositive is logically equivalent to the original while the inverse and converse are not.		
L4.3	Proof		
L4.3.1	Know the basic structure for the proof of an "If..., then..." statement (assuming the hypothesis and ending with the conclusion) and know that proving the contrapositive is equivalent.		
L4.3.2	Construct proofs by contradiction; use counterexamples, when appropriate, to disprove a statement.		

L4.3.3	Explain the difference between a necessary and a sufficient condition within the statement of a theorem; determine the correct conclusions based on interpreting a theorem in which necessary or sufficient conditions in the theorem or hypothesis are satisfied.		
2	Algebra & Functions		
A1	Students recognize, construct, interpret, and evaluate expressions. They fluently transform symbolic expressions into equivalent forms. They determine appropriate techniques for solving each type of equation, inequality, or system of equations, apply the techniques correctly to solve, justify the steps in the solutions, and draw conclusions from the solutions. They know and apply common formulas.		
A1.1	Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A1.1.1	Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.	Quadratics	Quadratic functions in the real world
A1.1.2	Know the definitions and properties of exponents and roots, transition fluently between them, and apply them in algebraic expressions.	Exponential and Logarithmic Functions	Properties of Logarithms
A1.1.3	Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).	Polynomials	Factoring Polynomials
		Polynomials	Working with Cubes
		Polynomials	The Factor Theorem
A1.1.4	Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., multiply $(x - 1)(1 - x^2 + 3)$; simplify $(9x - x^3)/(x + 3)$)	Polynomials	Combining polynomials
A1.1.5	Divide a polynomial by a monomial.	Polynomials	Dividing polynomials

A1.1.6	Transform exponential and logarithmic expressions into equivalent forms using the properties of exponents and logarithms including the inverse relationship between exponents and logarithms.	Exponential and Logarithm functions	Logarithm functions and exponential functions together
		Exponential and Logarithm functions	Properties of Logarithms
		Exponential and Logarithm functions	Values of logarithm functions
		Exponential and Logarithm functions	Computations with logarithm functions
		Exponential and Logarithm functions	Values of logarithm functions: a look at your calculator
		Exponential and Logarithm functions	Comparing sizes
		Exponential and Logarithm functions	Logarithm functions
		Exponential and Logarithm functions	Real Life Logarithmic Examples
		Exponential and Logarithm functions	More Real Life Logarithmic Examples
A1.1.7	Transform trigonometric expressions into equivalent forms using basic identities such as: $\sin^2 \theta + \cos^2 \theta = 1$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\tan^2 \theta + 1 = \sec^2 \theta$		
A1.2	Solutions of Equations and Inequalities (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A1.2.1	Write equations and inequalities with one or two variables to represent mathematical or applied situations, and solve.	Absolute Value	Absolute Value and Inequalities
		Absolute Value	Absolute Value and Inequalities Shortcuts Summary
A1.2.2	Associate a given equation with a function whose zeros are the solutions of the equation.	Composition of Functions	Function Notation
		Composition of Functions	Horizontal Line Test
		Composition of Functions	Review of Functions
		Composition of Functions	Definition of Functions

A1.2.3	Solve (and justify steps in the solutions) linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns; apply the quadratic formula appropriately.	Absolute Value	Absolute Value and Inequalities Shortcuts Summary
		Absolute Value	Absolute Value and Inequalities
		Systems of Linear Equations	Inconsistent Systems of Equations
		Systems of Linear Equations	Gauss - Jordan Elimination Method
		Systems of Linear Equations	Introduction
		Systems of Linear Equations	Underdetermined Systems of Equations
		Systems of Linear Equations	Using your calculator to solve systems of linear equations
		Systems of Linear Equations	Systems of two linear equations with two variables
		Systems of Linear Equations	Systems of Linear Inequalities
		Systems of Linear Equations	Systems having three linear equations
		Systems of Linear Equations	Substitution Method to solve a system
		Systems of Linear Equations	Addition Method of Solving Systems of Equations
		Systems of Linear Equations	Gauss - Jordan Elimination for systems with three equations and three variables
		Quadratics	Introduction
		Quadratics	The Quadratic Formula
		Quadratics	Developing the Quadratic Formula
A1.2.4	Solve absolute value equations and inequalities, (e.g. solve $ x - 3 $ "is less than or equal to 6), and justify steps in the solution.	Absolute Value	Absolute Value equations in other places
		Absolute Value	Absolute Value Equations
		Absolute Value	Absolute Value and Inequalities
		Absolute Value	More Complicated Absolute Value Equations

		Absolute Value	Absolute Value and Inequalities Shortcuts
		Absolute Value	Shortcuts
		Absolute Value	Absolute Value and Inequalities Shortcuts Summary
A1.2.5	Solve polynomial equations and equations involving rational expressions (e.g. solve $-2x(x^2 + 4x+3) = 0$; solve $x - (1/(x + 6)) = 3$), and justify steps in the solution.	Quadratics	Zeros of the quadratic function
A1.2.6	Solve power equations (e.g., $(x + 1)^3 = 8$) and equations including radical expressions (e.g., the square root of $(3x - 7) = 7$), justify steps in the solution, and explain how extraneous solutions may arise.		
A1.2.7	Solve exponential and logarithmic equations (e.g., $3(2 \text{ to the } x \text{ power}) = 24$), $2 \ln(x + 1) = 4$), and justify steps in the solution.	Exponential and Logarithm functions	Graphs of logarithm functions
		Exponential and Logarithm functions	The horizontal line property, and one-to-one functions
A1.2.8	Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable, and justify steps in the solution.	Systems of Linear Equations	Substitution method to solve a system
A1.2.9	Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = velocity x time), and apply appropriately in contextual situations.	Quadratics	Developing the Quadratic Formula
		Quadratics	The Quadratic Formula
		Conic Sections	The Distance Formula
A1.2.10	Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals (e.g., $2\sin x - 1 = 0$ for $0 \leq x \leq 2\pi$).		
A2	Students understand functions, their representations, and their attributes. They perform transformations, combine and compose functions, and find inverses. Students classify functions and know the characteristics of each family. They work with functions with real coefficients fluently.		

A2.1	Definitions, Representations, and Attributes of Functions		
A2.1.1	Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function; and identify its domain and range.	Composition of Functions	Function Notation
		Composition of Functions	Inverse functions
		Composition of Functions	Review of Functions
		Composition of Functions	Horizontal Line Test
		Composition of Functions	Definition of Functions
		Composition of Functions	Domain Restrictions
A2.1.2	Read, interpret, and use function notation, and evaluate a function at a value in its domain.	Composition of Functions	Definition of Functions
		Composition of Functions	Domain Restrictions
		Composition of Functions	Function Notation
		Composition of Functions	Checking that two functions really are inverse functions of each other
		Composition of Functions	Review of Functions
		Composition of Functions	Horizontal Line Test
A2.1.3	Represent functions in symbols, graphs, tables, diagrams, or words, and translate among representations.	Composition of Functions	Domain Restrictions
		Composition of Functions	Function Notation
		Composition of Functions	Inverse functions
		Composition of Functions	Horizontal Line Test
A2.1.4	Recognize that functions may be defined by different expressions over different intervals of their domains; such functions are piecewise-defined (e.g., absolute value and greatest integer functions).		

A2.1.5	Recognize that functions may be defined recursively, and compute values of and graph simple recursively defined functions (e.g., $f(0) = 5$, and $f(n) = f(n-1) + 2$).		
A2.1.6	Identify the zeros of a function and the intervals where the values of a function are positive or negative, and describe the behavior of a function, as x approaches positive or negative infinity, given the symbolic and graphical representations.		
A2.1.7	Identify and interpret the key features of a function from its graph or its formula(e), (e.g. slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, average rate of change over an interval, and periodicity).	Composition of Functions	Domain Restrictions
		Composition of Functions	Horizontal Line Test
A2.2	Operations and Transformations		
A2.2.1	Combine functions by addition, subtraction, multiplication, and division.	Composition of Functions	Combining functions
A2.2.2	Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the x - and y -axes) to basic functions, and represent symbolically.		
A2.2.3	Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs (e.g., $f(x) = x^3$ and $g(x) = x$ to the $1/3$ power).	Composition of Functions	Function Notation
		Composition of Functions	Inverse functions
		Composition of Functions	Checking that two functions really are inverse functions of each other
		Composition of Functions	Finding an Inverse Function
		Conic Sections	Parabolas in Standard Form
A2.2.4	If a function has an inverse, find the expression(s) for the inverse.	Composition of Functions	Finding an Inverse Function
A2.2.5	Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.	Composition of Functions	Composition of functions

A2.2.6	Know and interpret the function notation for inverses and verify that two functions are inverses using composition.	Composition of Functions	Checking that two functions really are inverse functions of each other
A2.3	Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)		
A2.3.1	Identify a function as a member of a family of functions based on its symbolic, or graphical representation; recognize that different families of functions have different asymptotic behavior at infinity, and describe these behaviors.	Composition of Functions	Horizontal Line Test
		Composition of Functions	Definition of Functions
		Composition of Functions	Domain Restrictions
		Composition of Functions	Function Notation
		Composition of Functions	Inverse functions
A2.3.2	Describe the tabular pattern associated with functions having constant rate of change (linear); or variable rates of change.		
A2.3.3	Write the general symbolic forms that characterize each family of functions. (e.g., $f(x) = A$ to the base 0 of a to the x power; $f(x) = A \sin Bx$)		
A2.4	Lines and Linear Functions		
A2.4.1	Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$, where B "is not equal to" 0], point-slope, and slope-intercept) given appropriate information, and convert between forms.	Systems of Linear Equations	Introduction
A2.4.2	Graph lines (including those of the form $x = h$ and $y = k$) given appropriate information.		
A2.4.3	Relate the coefficients in a linear function to the slope and x - and y -intercepts of its graph.	Systems of Linear Equations	Introduction
A2.4.4	Find an equation of the line parallel or perpendicular to given line, through a given point; understand and use the facts that non-vertical parallel lines have equal slopes, and that non-vertical perpendicular lines have slopes that multiply to give -1.		

A2.5	Exponential and Logarithmic Functions		
A2.5.1	Write the symbolic form and sketch the graph of an exponential function given appropriate information. (e.g., given an initial value of 4 and a rate of growth of 1.5, write $f(x) = 4(1.5)^x$).	Composition of Functions	Horizontal Line Test
		Exponential and Logarithm functions	Exponential functions: an intuitive approach
		Exponential and Logarithm functions	Introduction
		Exponential and Logarithm functions	Computations with exponential functions
		Exponential and Logarithm functions	Exponential functions: an example
		Exponential and Logarithm functions	Graphs of exponential functions
A2.5.2	Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^x$, $f(x) = \log x$, $f(x) = e^x$, $f(x) = \ln x$); recognize the logarithmic function as the inverse of the exponential function.	Exponential and Logarithm functions	Graphs of exponential functions
		Exponential and Logarithm functions	Exponential functions: an example
		Exponential and Logarithm functions	The Natural Logarithm function
		Exponential and Logarithm functions	Exponential functions: an intuitive approach
		Exponential and Logarithm functions	Graphs of logarithm functions
		Exponential and Logarithm functions	Exponential functions: the formal definition
		Exponential and Logarithm functions	Computations with exponential functions
		Exponential and Logarithm functions	The horizontal line property, and one-to-one functions
		Exponential and Logarithm functions	Introduction
		Composition of Functions	Horizontal Line Test
		Composition of Functions	Inverse functions

		Composition of Functions	Finding an Inverse Function
		Composition of Functions	Checking that two functions really are inverse functions of each other
A2.5.3	Apply properties of exponential and logarithmic functions (e.g., a to the (x+y) power = a to the x power times a to the y power; $\log(ab) = \log a + \log b$).	Exponential and Logarithm functions	Properties of Logarithms
		Exponential and Logarithm functions	Computations with exponential functions
		Exponential and Logarithm functions	Exponential functions: an example
		Exponential and Logarithm functions	Graphs of exponential functions
A2.5.4	Understand and use the fact that the base of an exponential function determines whether the function increases or decreases and understand how the base affects the rate of growth or decay.	Exponential and Logarithm functions	Exponential functions: an example
		Exponential and Logarithm functions	Computations with exponential functions
		Exponential and Logarithm functions	Exponential functions: an intuitive approach
		Exponential and Logarithm functions	Graphs of exponential functions
		Exponential and Logarithm functions	Exponential functions: the formal definition
		Exponential and Logarithm functions	Introduction
		Exponential and Logarithm functions	Exponential functions with fractional bases
A2.5.5	Relate exponential and logarithmic functions to real phenomena, including half-life and doubling time.	Composition of Functions	Function Notation
		Exponential and Logarithm functions	Exponential functions: the formal definition
		Exponential and Logarithm functions	Introduction
		Exponential and Logarithm functions	Computations with exponential functions
		Exponential and Logarithm functions	Exponential functions: an example
		Exponential and Logarithm functions	Graphs of exponential functions

		Exponential and Logarithm functions	Exponential functions: an intuitive approach
A2.6	Quadratic Functions		
A2.6.1	Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).	Conic Sections	Parabolas in Standard Form
		Quadratics	Quadratic functions and their graphs
		Quadratics	From the zeros to the equation of quadratic functions
		Quadratics	Quadratic functions in the real world
		Quadratics	Zeros of the quadratic function
		Quadratics	Introduction
		Quadratics	Factored form of quadratics
		Composition of Functions	Horizontal Line Test
		Composition of Functions	Domain Restrictions
A2.6.2	Identify the elements of a parabola (vertex, axis of symmetry, direction of opening) given its symbolic form or its graph, and relate these elements to the coefficient(s) of the symbolic form of the function.	Conic Sections	Parabolas
		Conic Sections	Parabolas in Standard Form
		Quadratics	Quadratic functions and their graphs
		Quadratics	Graphing Parabolas
A2.6.3	Convert quadratic functions from standard to vertex form by completing the square.	Quadratics	Introduction
		Quadratics	Zeros of the quadratic function
		Quadratics	Quadratic functions and their graphs
		Quadratics	Factored form of quadratics
		Quadratics	Completing the Square
		Quadratics	From the zeros to the equation of quadratic functions
		Quadratics	Quadratic functions in the real world

		Conic Sections	Parabolas in Standard Form
		Conic Sections	Parabolas
A2.6.4	Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.	Quadratics	The Discriminant of a Quadratic
		Quadratics	Quadratic functions and their graphs
		Quadratics	From the zeros to the equation of quadratic functions
		Quadratics	Quadratic functions in the real world
		Quadratics	Zeros of the quadratic function
		Quadratics	Introduction
		Quadratics	Factored form of quadratics
		Composition of Functions	Horizontal Line Test
		Composition of Functions	Domain Restrictions
A2.6.5	Express quadratic functions in vertex form to identify their maxima or minima, and in factored form to identify their zeros.	Quadratics	Zeros of the quadratic function
		Quadratics	Introduction
		Quadratics	Factored form of quadratics
		Quadratics	Quadratic functions and their graphs
		Quadratics	From the zeros to the equation of quadratic functions
		Quadratics	Quadratic functions in the real world
		Conic Sections	Parabolas
		Conic Sections	Parabolas in Standard Form
A2.7	Power Functions (including roots, cubics, quartics, etc.)		
A2.7.1	Write the symbolic form and sketch the graph of power functions.		

A2.7.2	Express direct and inverse relationships as functions (e.g., $y = kx$ to the n power and $y = kx$ to the $-n$ power, $n > 0$) and recognize their characteristics (e.g., in $y = x^3$, note that doubling x results in multiplying y by a factor of 8).		
A2.7.3	Analyze the graphs of power functions, noting reflectional or rotational symmetry.		
A2.8	Polynomial Functions		
A2.8.1	Write the symbolic form and sketch the graph of simple polynomial functions.	Composition of Functions	Horizontal Line Test
		Composition of Functions	Domain Restrictions
A2.8.2	Understand the effects of degree, leading coefficient, and number of real zeros on the graphs of polynomial functions of degree greater than 2.		
A2.8.3	Determine the maximum possible number of zeros of a polynomial function, and understand the relationship between the x -intercepts of the graph and the factored form of the function.		
A2.9	Rational Functions		
A2.9.1	Write the symbolic form and sketch the graph of simple rational functions.		
A2.9.2	Analyze graphs of simple rational functions (e.g., $f(x) = (2x + 1)/(x - 1)$; $g(x) = x/(x^2 - 4)$) and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.		
A2.10	Trigonometric Functions		
A2.10.1	Use the unit circle to define sine and cosine; approximate values of sine and cosine (e.g., $\sin 3$, or $\cos 0.5$); use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.		
A2.10.2	Use the relationship between degree and radian measures to solve problems.		
A2.10.3	Use the unit circle to determine the exact values of sine and cosine, for integer multiples of $\pi/6$ and $\pi/4$.		

A2.10.4	Graph the sine and cosine functions; analyze graphs by noting domain, range, period, amplitude, and location of maxima and minima.		
A2.10.5	Graph transformations of basic trigonometric functions (involving changes in period, amplitude, and midline) and understand the relationship between constants in the formula and the transformed graph.		
A3	Students construct or select a function to model a real-world situation in order to solve applied problems. They draw on their knowledge of families of functions to do so.		
A3.1	Models of Real-world Situations Using Families of Functions.		
A3.1.1	Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. In the example above, recognize that the appropriate general function is exponential ($P = P_0 a^{kt}$))		
A3.1.2	Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. In the example above, substitute the given values $P_0 = 300$ and $a = 1.02$ to obtain $P = 300(1.02)^{kt}$.	Exponential and Logarithmic Functions	Computations with exponential functions
A3.1.3	Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.	Composition of Functions	Function Notation
A3.1.4	Use methods of linear programming to represent and solve simple real-life problems.		

3	Geometry & Trigonometry		
G1	Students represent basic geometric figures, polygons, and conic sections and apply their definitions and properties in solving problems and justifying arguments, including constructions and representations in the coordinate plane. Students represent three-dimensional figures, understand the concepts of volume and surface area, and use them to solve problems. They know and apply properties of common three-dimensional figures.		
G1.1	Lines and Angles; Basic Euclidean and Coordinate Geometry		
G1.1.1	Solve multi-step problems and construct proofs involving vertical angles, linear pairs of angles supplementary angles, complementary angles, and right angles.		
G1.1.2	Solve multi-step problems and construct proofs involving corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.		
G1.1.3	Perform and justify constructions, including midpoint of a line segment and bisector of an angle, using straightedge and compass.		
G1.1.4	Given a line and a point, construct a line through the point that is parallel to the original line using straightedge and compass; given a line and a point, construct a line through the point that is perpendicular to the original line; justify the steps of the constructions.		
G1.1.5	Given a line segment in terms of its endpoints in the coordinate plane, determine its length and midpoint.		
G1.1.6	Recognize Euclidean Geometry as an axiom system; know the key axioms and understand the meaning of and distinguish between undefined terms (e.g., point, line, plane), axioms, definitions, and theorems.		

G1.2	Triangles and Their Properties		
G1.2.1	Prove that the angle sum of a triangle is 180° and that an exterior angle of a triangle is the sum of the two remote interior angles.		
G1.2.2	Construct and justify arguments and solve multi-step problems involving angle measure, side length, perimeter, and area of all types of triangles.		
G1.2.3	Know a proof of the Pythagorean Theorem and use the Pythagorean Theorem and its converse to solve multi-step problems.		
G1.2.4	Prove and use the relationships among the side lengths and the angles of 30° - 60° - 90° triangles and 45° - 45° - 90° triangles.		
G1.2.5	Solve multi-step problems and construct proofs about the properties of medians, altitudes, and perpendicular bisectors to the sides of a triangle, and the angle bisectors of a triangle; using a straightedge and compass, construct these lines.		
G1.3	Triangles and Trigonometry		
G1.3.1	Define the sine, cosine, and tangent of acute angles in a right triangle as ratios of sides; solve problems about angles, side lengths, or areas using trigonometric ratios in right triangles.		
G1.3.2	Know and use the Law of Sines and the Law of Cosines and use them to solve problems; find the area of a triangle with sides a and b and included angle θ using the formula $\text{Area} = (1/2) a b \sin \theta$.		
G1.3.3	Determine the exact values of sine, cosine, and tangent for 0° , 30° , 45° , 60° , and their integer multiples, and apply in various contexts.		
G1.4	Quadrilaterals and Their Properties		
G1.4.1	Solve multi-step problems and construct proofs involving angle measure, side length, diagonal length, perimeter, and area of squares, rectangles, parallelograms, kites, and trapezoids.		

G1.4.2	Solve multi-step problems and construct proofs involving quadrilaterals (e.g., prove that the diagonals of a rhombus are perpendicular) using Euclidean methods or coordinate geometry.		
G1.4.3	Describe and justify hierarchical relationships among quadrilaterals, (e.g. every rectangle is a parallelogram).		
G1.4.4	Prove theorems about the interior and exterior angle sums of a quadrilateral.		
G1.4.5	Understand the definition of a cyclic quadrilateral and know and use the basic properties of cyclic quadrilaterals.		
G1.5	Other Polygons and Their Properties		
G1.5.1	Know and use subdivision or circumscription methods to find areas of polygons (e.g., regular octagon, non-regular pentagon).		
G1.5.2	Know, justify, and use formulas for the perimeter and area of a regular n-gon and formulas to find interior and exterior angles of a regular n-gon and their sums.		
G1.6	Circles and Their Properties		
G1.6.1	Solve multi-step problems involving circumference and area of circles.		
G1.6.2	Solve problems and justify arguments about chords (e.g., if a line through the center of a circle is perpendicular to a chord, it bisects the chord) and lines tangent to circles (e.g., a line tangent to a circle is perpendicular to the radius drawn to the point of tangency).		
G1.6.3	Solve problems and justify arguments about central angles, inscribed angles and triangles in circles.		
G1.6.4	Know and use properties of arcs and sectors, and find lengths of arcs and areas of sectors.		
G1.7	Conic Sections and Their Properties		
G.1.7.1	Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.	Conic Sections	Circles in Standard Form
		Conic Sections	Finding the Center and Radius of a Circle
		Conic Sections	Circles

G1.7.2	Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.	Quadratics	Quadratic functions and their graphs
		Conic Sections	Circles
		Conic Sections	Ellipses in Standard Form
		Conic Sections	How to get the equation of an ellipse
		Conic Sections	General equation for conic sections.
		Conic Sections	Introduction
		Conic Sections	Circles in Standard Form
		Conic Sections	Foci of Ellipses
		Conic Sections	Hyperbolas
		Conic Sections	What kind of conic is it?
		Conic Sections	Parabolas
		Conic Sections	Finding the Center and Radius of a Circle
		Conic Sections	"Vertical" ellipses
		Conic Sections	Hyperbola in Standard Form
		Conic Sections	Parabolas in Standard Form
		Conic Sections	Ellipses
		Conic Sections	Eccentricity
		Conic Sections	Asymptotes for hyperbolas
G1.7.3	Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.	Conic Sections	"Vertical" ellipses
		Conic Sections	Hyperbola in Standard Form
		Conic Sections	Ellipses
		Conic Sections	Eccentricity
		Conic Sections	Asymptotes for hyperbolas
		Conic Sections	Ellipses in Standard Form
		Conic Sections	How to get the equation of an ellipse
		Conic Sections	Foci of Ellipses
		Conic Sections	Hyperbolas

G1.7.4	Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola; interpret these relationships in applied contexts.	Conic Sections	Parabolas in Standard Form
		Conic Sections	Foci of Ellipses
		Conic Sections	Asymptotes for hyperbolas
G1.8	Three- Dimensional Figures		
G1.8.1	Solve multi-step problems involving surface area and volume of pyramids, prisms, cones, cylinders, hemispheres, and spheres.		
G1.8.2	Identify symmetries of pyramids, prisms, cones, cylinders, hemispheres, and spheres.		
G2	Students use and justify relationships between lines, angles, area and volume formulas, and 2- and 3-dimensional representations. They solve problems and provide proofs about congruence and similarity.		
G2.1	Relationships Between Area and Volume Formulas		
G2.1.1	Know and demonstrate the relationships between the area formula of a triangle, the area formula of a parallelogram, and the area formula of a trapezoid.		
G2.1.2	Know and demonstrate the relationships between the area formulas of various quadrilaterals (e.g., explain how to find the area of a trapezoid based on the areas of parallelograms and triangles).		
G2.1.3	Know and use the relationship between the volumes of pyramids and prisms (of equal base and height) and cones and cylinders (of equal base and height).		
G2.2	Relationships Between Two-dimensional and Three-dimensional Representations		
G2.2.1	Identify or sketch a possible 3-dimensional figure, given 2-dimensional views (e.g., nets, multiple views); create a 2-dimensional representation of a 3-dimensional figure.		

G2.2.2	Identify or sketch cross-sections of 3-dimensional figures; identify or sketch solids formed by revolving 2-dimensional figures around lines.		
G2.3	Congruence and Similarity		
G2.3.1	Prove that triangles are congruent using the SSS, SAS, ASA, and AAS criteria, and for right triangles, the hypotenuse-leg criterion.		
G2.3.2	Use theorems about congruent triangles to prove additional theorems and solve problems, with and without use of coordinates.		
G2.3.3	Prove that triangles are similar by using SSS, SAS, and AA conditions for similarity.		
G2.3.4	Use theorems about similar triangles to solve problems with and without use of coordinates.		
G2.3.5	Know and apply the theorem stating that the effect of a scale factor of k relating one two dimensional figure to another or one three dimensional figure to another, on the length, area, and volume of the figures is to multiply each by k , k^2 , and k^3 , respectively.		
G3	Students will solve problems about distance-preserving transformations and shape-preserving transformations. The transformations will be described synthetically and, in simple cases, by analytic expressions in coordinates.		
G3.1	Distance-preserving Transformations: Isometries		
G3.1.1	Define reflection, rotation, translation, and glide reflection and find the image of a figure under a given isometry.		
G3.1.2	Given two figures that are images of each other under an isometry, find the isometry and describe it completely.		
G3.1.3	Find the image of a figure under the composition of two or more isometries, and determine whether the resulting figure is a reflection, rotation, translation, or glide reflection image of the original figure.		
G3.2	Shape-preserving Transformations: Dilations and Isometries		

G3.2.1	Know the definition of dilation, and find the image of a figure under a given dilation.		
G3.2.2	Given two figures that are images of each other under some dilation, identify the center and magnitude of the dilation.		
G3.2.3	Find the image of a figure under the composition of a dilation and an isometry.		
4	Statistics & Probability		
S1	Students plot and analyze univariate data by considering the shape of distributions and analyzing outliers; they find and interpret commonly-used measures of center and variation; and they explain and use properties of the normal distribution.		
S1.1	Producing and Interpreting Plots		
S1.1.1	Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.		
S1.1.2	Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.		
S1.2	Measures of Center and Variation		
S1.2.1	Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.		
S1.2.2	Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.		
S1.2.3	Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.		

S1.3	The Normal Distribution		
S1.3.1	Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.		
S1.3.2	Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.		
S1.3.3	Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.		
S1.3.4	Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.		
S2	Students plot and interpret bivariate data by constructing scatterplots, recognizing linear and nonlinear patterns, and interpreting correlation coefficients; they fit and interpret regression models, using technology as appropriate.		
S2.1	Scatterplots and Correlation		
S2.1.1	Construct a scatterplot for a bivariate data set with appropriate labels and scales.		
S2.1.2	Given a scatterplot, identify patterns, clusters, and outliers; recognize no correlation, weak correlation, and strong correlation.		
S2.1.3	Estimate and interpret Pearson's correlation coefficient for a scatterplot of a bivariate data set; recognize that correlation measures the strength of linear association.		
S2.1.4	Differentiate between correlation and causation; know that a strong correlation does not imply a cause-and-effect relationship; recognize the role of lurking variables in correlation.		
S2.2	Linear Regression		
S2.2.1	For bivariate data which appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line; interpret the slope of the equation for a regression line.		

S2.2.2	Use the equation of the least squares regression line to make appropriate predictions.		
S3	Students understand and apply sampling and various sampling methods, examine surveys and experiments, identify bias in methods of conducting surveys, and learn strategies to minimize bias. They understand basic principles of good experimental design.		
S3.1	Data Collection and Analysis		
S3.1.1	Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.		
S3.1.2	Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.		
S3.1.3	Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.		
S3.1.4	Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.		
S3.1.5	Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.		
S3.1.6	Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.		
S3.2.1	Explain the basic ideas of statistical process control, including recording data from a process over time.		
S3.2.2	Read and interpret basic control charts; detect patterns and departures from patterns.		

S4	Students understand probability and find probabilities in various situations, including those involving compound events, using diagrams, tables, geometric models and counting strategies; they apply the concepts of probability to make decisions.		
S4.1	Probability		
S4.1.1	Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).	Counting	Frequency Expectation Interpretation of probability
S4.1.2	Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.	Counting	Probability: More examples
		Counting	Frequency Expectation Interpretation of probability
S4.1.3	Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.	Counting	Frequency Expectation Interpretation of probability
S4.2	Application and Representation		
S4.2.1	Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.	Counting	Factorial notation
		Counting	Permutations
		Counting	Frequency Expectation Interpretation of probability
		Counting	Some computations with factorials
		Counting	Counting: An introduction to choosing subsets
		Counting	Counting: An introduction to the Multiplication Principle
		Counting	Combinations
		Counting	More about the Multiplication Principle
		Counting	Counting Subsets Formula



S4.2.2	Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.	Counting	Introduction
		Counting	Probability: An introduction
		Counting	Probability: More examples
		Counting	Frequency Expectation Interpretation of probability